

## Overview of YEAR 13 AUTUMN 2

Week	Statements	Teaching activities	Notes
1	<p>1.08a Know and be able to use the fundamental theorem of calculus.</p> <p><i>i.e. Learners should know that integration may be defined as the reverse of differentiation and be able to apply the result that <math>\int f(x) dx = F(x) + c \Leftrightarrow f(x) = \frac{d}{dx}(F(x))</math> for sufficiently well-behaved functions.</i></p> <p><i>Includes understanding and being able to use the terms indefinite and definite when applied to integrals.</i></p>		<p>CHAPTER 11 FURTHER INTEGRATION TECHNIQUES</p> <p>SECTION 1 REVERSING STANDARD DERIVATIVES Page 219</p> <p>EXERCISE 11A Page 222-223</p>
	<p>1.07i Be able to differentiate <math>x^n</math> for rational values of <math>n</math> and related constant multiples, sums and differences.</p>		
	<p>1.07j Be able to differentiate <math>e^{kx}</math> and <math>a^{kx}</math> and related sums, differences and constant multiples.</p>		

	1.07k Be able to differentiate $\sin kx$ , $\cos kx$ , $\tan kx$ and related sums, differences and constant multiples.		
	1.07l Understand and be able to use the derivative of $\ln x$ .		

Week	Statements	Teaching activities	Notes
2	<p>1.08h Be able to carry out simple cases of integration by substitution.</p> <p><i>Learners should understand the relationship between this method and the chain rule.</i></p> <p><i>Learners will be expected to integrate examples in the form <math>f'(x)(f(x))^n</math>, such as <math>(2x + 3)^5</math> or <math>x(x^2 + 3)^7</math>, either by inspection or substitution.</i></p> <p><i>Learners will be expected to recognise an integrand of the form <math>\frac{f'(x)}{f(x)}</math> such as <math>\frac{x^2+x}{2x^3+3x^2-7}</math> or <math>\tan x</math>.</i></p>		<p>SECTION 2 INTEGRATION BY SUBSTITUTION Page 223</p> <p>Reversing the Chain Rule</p> <p>EXERCISE 11B Page 227</p>

	<p><i>Integration by substitution is limited to cases where one substitution will lead to a function which can be integrated. Substitutions may or may not be given.</i></p> <p><i>Learners should be able to find a suitable substitution in integrands such as <math>\frac{(4x-1)}{(2x+1)^5}</math>, <math>\sqrt{9-x^2}</math> or <math>\frac{1}{1+\sqrt{x}}</math>.</i></p>		
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<b>Week</b>	<b>Statements</b>	<b>Teaching activities</b>	<b>Notes</b>
3	<p>1.08h Be able to carry out simple cases of integration by substitution.</p> <p><i>Learners should understand the relationship between this method and the chain rule.</i></p> <p><i>Learners will be expected to integrate examples in the form <math>f'(x)(f(x))^n</math>, such as <math>(2x+3)^5</math> or <math>x(x^2+3)^7</math>, either by inspection or substitution.</i></p> <p><i>Learners will be expected to recognise an integrand of the form <math>\frac{kf'(x)}{f(x)}</math> such as <math>\frac{x^2+x}{2x^3+3x^2-7}</math> or <math>\tan x</math>.</i></p> <p><i>Integration by substitution is limited to cases where one substitution will lead</i></p>		<p>GENERAL SUBSTITUTION Page 227</p> <p>EXERCISE 11C Page 229-230</p>

	<p><i>to a function which can be integrated. Substitutions may or may not be given.</i></p> <p><i>Learners should be able to find a suitable substitution in integrands such as <math>\frac{(4x-1)}{(2x+1)^5}</math>, <math>\sqrt{9-x^2}</math> or <math>\frac{1}{1+\sqrt{x}}</math>.</i></p>		
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<b>Week</b>	<b>Statements</b>	<b>Teaching activities</b>	<b>Notes</b>
4	<p>1.08i Be able to carry out simple cases of integration by parts.</p> <p><i>Learners should understand the relationship between this method and the product rule.</i></p> <p><i>Integration by parts may include more than one application of the method e.g. <math>x^2 \sin x</math>.</i></p> <p><i>Learners will be expected to be able to apply integration by parts to the integral of <math>\ln x</math> and related functions.</i></p> <p><i>[Reduction formulae are excluded.]</i></p>		<p>SECTION 3 INTEGRATION BY PARTS Page 230</p> <p>EXERCISE 11D Page 232</p>

<b>Week</b>	<b>Statements</b>	<b>Teaching activities</b>	<b>Notes</b>
5	<p>1.08i Be able to carry out simple cases of integration by parts.</p> <p><i>Learners should understand the relationship between this method and the product rule.</i></p> <p><i>Integration by parts may include more than one application of the method e.g. <math>x^2 \sin x</math>.</i></p> <p><i>Learners will be expected to be able to apply integration by parts to the integral of <math>\ln x</math> and related functions.</i></p> <p><i>[Reduction formulae are excluded.]</i></p>		<p>REPEATED INTEGRATION BY PARTS Page 233</p> <p>EXERCISE 11E Page 234</p>

<b>Week</b>	<b>Statements</b>	<b>Teaching activities</b>	<b>Notes</b>
6	<p>1.05I Understand and be able to use double angle formulae and the formulae for <math>\sin(A \pm B)</math>, <math>\cos(A \pm B)</math> and <math>\tan(A \pm B)</math>.</p> <p><i>Learners may be required to use the formulae to prove trigonometric identities, simplify expressions, evaluate expressions exactly, solve</i></p>		<p>SECTION 4 USING TRIGONOMETRIC IDENTITIES IN INTEGRATION Page 234</p> <p>EXERCISE 11F Page 238-239</p>

	<i>trigonometric equations or find derivatives and integrals.</i>		
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<b>Week</b>	<b>Statements</b>	<b>Teaching activities</b>	<b>Notes</b>
7	<p>1.05I Understand and be able to use double angle formulae and the formulae for <math>\sin(A \pm B)</math>, <math>\cos(A \pm B)</math> and <math>\tan(A \pm B)</math>.</p> <p><i>Learners may be required to use the formulae to prove trigonometric identities, simplify expressions, evaluate expressions exactly, solve trigonometric equations or find derivatives and integrals.</i></p>		<p>SECTION 4 USING TRIGONOMETRIC IDENTITIES CONTINUED Page 234</p> <p>EXERCISE 11F Page 238-239</p>

<b>Week</b>	<b>Statements</b>	<b>Teaching activities</b>	<b>Notes</b>
8	<p>1.08j Be able to integrate functions using partial fractions that have linear terms in the denominator.</p> <p><i>i.e. Functions with denominators no more complicated than the forms <math>(ax + b)(cx + d)^2</math> or <math>(ax + b)(cx + d)(ex + f)</math>.</i></p>		<p>SECTION 5 INTEGRATING RATIONAL FUNCTIONS Page 239</p> <p>EXERCISE 11G Page 242</p>

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<b>Week</b>	<b>Statements</b>	<b>Teaching activities</b>	<b>Notes</b>
9			MIXED PRACTICE 11 Page 242-243

<b>Week</b>	<b>Statements</b>	<b>Teaching activities</b>	<b>Notes</b>
10	1.07p Be able to apply differentiation to find points of inflection on a curve. In particular, learners should know that if a curve has a point of inflection at $x$ then $f''(x) = 0$ and there is a sign change in the second derivative on either side of $x$ ; if also $f'(x) = 0$ at that point, then the point of inflection is a stationary point, but if $f'(x) \neq 0$ at that point, then the point of inflection is not a stationary point.		CHAPTER 12 FURTHER APPLICATIONS OF CALCULUS Page 246  SECTION 1 PROPERTIES OF CURVES Page 247  EXERCISE 12 A Page 252

<b>Week</b>	<b>Statements</b>	<b>Teaching activities</b>	<b>Notes</b>
11	1.03g Understand and be able to use the parametric equations of curves and		SECTION 2 PARAMETRIC EQUATIONS Page 253

	<p>be able to convert between cartesian and parametric forms.</p> <p><i>Learners should understand the meaning of the terms parameter and parametric equation.</i></p> <p><i>Includes sketching simple parametric curves.</i></p> <p><i>See also Section 1.07s.</i></p>		
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<b>Week</b>	<b>Statements</b>	<b>Teaching activities</b>	<b>Notes</b>
12	<p>1.03h Be able to use parametric equations in modelling in a variety of contexts.</p> <p><i>The contexts may be within pure mathematics or in realistic contexts, for example those involving related rates of change.</i></p>		<p>PARAMETRIC EQUATIONS IN CONTEXT Page 253</p> <p>EXERCISE 12B Page 256-257</p>

<b>Week</b>	<b>Statements</b>	<b>Teaching activities</b>	<b>Notes</b>
13	<p>1.07s Be able to differentiate simple functions and relations defined</p>		<p>DIFFERENTIATING PARAMETRIC EQUATIONS Page 258</p>



	<p>implicitly or parametrically for the first derivative only.</p> <p><i>They should be able to find the gradient at a point on a curve and to use this to find the equations of tangents and normals, and to solve associated problems.</i></p> <p><i>Includes differentiation of functions defined in terms of a parameter using the chain rule.</i></p>		EXERCISE 12C Page 262
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<b>Week</b>	<b>Statements</b>	<b>Teaching activities</b>	<b>Notes</b>
14			INTEGRATING PARAMETRIC EQUATIONS Page 263  EXERCISE 12D Page 264-265

<b>Week</b>	<b>Statements</b>	<b>Teaching activities</b>	<b>Notes</b>
15	1.07t Be able to construct simple differential equations in pure mathematics and in context (contexts may include kinematics, population		SECTION 3 RELATED RATES OF CHANGE Page 266  EXERCISE 12E Page 268

	growth and modelling the relationship between price and demand).		
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<b>Week</b>	<b>Statements</b>	<b>Teaching activities</b>	<b>Notes</b>
16	<p>1.08f Be able to use a definite integral to find the area between two curves.</p> <p><i>This may include using integration to find the area of a region bounded by a curve and lines parallel to the coordinate axes, or between two curves or between a line and a curve.</i></p> <p><i>This includes curves defined parametrically.</i></p>		<p>SECTION 4 MORE COMPLICATED AREAS Page 269</p> <p>EXERCISE 12F Page 274-276</p>

<b>Week</b>	<b>Statements</b>	<b>Teaching activities</b>	<b>Notes</b>
17			MIXED PRACTICE 12 Page 278-281

<b>Week</b>	<b>Statements</b>	<b>Teaching activities</b>	<b>Notes</b>
18			MIXED PRACTICE 12 CONTINUED Page 278-281

<b>Week</b>	<b>Statements</b>	<b>Teaching activities</b>	<b>Notes</b>
19	1.07t Be able to construct simple differential equations in pure mathematics and in context (contexts may include kinematics, population growth and modelling the relationship between price and demand).		CHAPTER 13 DIFFERENTIAL EQUATIONS Page 282  SECTION 1 INTRODUCTION TO DIFFERENTIAL EQUATIONS Page 283  EXERCISE 13A Page 284

<b>Week</b>	<b>Statements</b>	<b>Teaching activities</b>	<b>Notes</b>
20	1.07t Be able to construct simple differential equations in pure mathematics and in context (contexts may include kinematics, population growth and modelling the relationship between price and demand).		SECTION 2 SEPARABLE DIFFERENTIAL EQUATIONS Page 285  EXERCISE 13B Page 288

<b>Week</b>	<b>Statements</b>	<b>Teaching activities</b>	<b>Notes</b>
21	1.07t Be able to construct simple differential equations in pure mathematics and in context (contexts may include kinematics, population growth and modelling the relationship between price and demand).		SECTION 3 MODELLING WITH DIFFERENTIAL EQUATIONS Page 289  EXERCISE 13C Page 292-294

<b>Week</b>	<b>Statements</b>	<b>Teaching activities</b>	<b>Notes</b>
22	1.07t Be able to construct simple differential equations in pure mathematics and in context (contexts may include kinematics, population growth and modelling the relationship between price and demand).		MIXED PRACTICE 13 Page 295-296

<b>Week</b>	<b>Statements</b>	<b>Teaching activities</b>	<b>Notes</b>
23	1.09a Be able to locate roots of $f(x) = 0$ by considering changes of sign of $f(x)$ in an interval of $x$ on which $y = f(x)$ is sufficiently well-behaved.  <i>Includes verifying the level of accuracy</i>		CHAPTER 14 NUMERICAL SOLUTION OF EQUATIONS Page 297  SECTION 1 LOCATING ROOTS OF A FUNCTION-CHANGE OF SIGN Page 299

	<i>of an approximation by considering upper and lower bounds.</i>		EXERCISE 14A Page 302
	1.09b Understand how change of sign methods can fail.  <i>e.g. when the curve <math>y = f(x)</math> touches the x-axis or has a vertical asymptote.</i>		

<b>Week</b>	<b>Statements</b>	<b>Teaching activities</b>	<b>Notes</b>
24	1.09c Be able to solve equations approximately using simple iterative methods, and be able to draw associated cobweb and staircase diagrams.		SECTION 2 THE NEWTON-RAPHSON METHOD Page 304  EXERCISE 14B Page 307
	1.09d Be able to solve equations using the Newton-Raphson method and other recurrence relations of the form $x_{n+1} = g(x_n)$ .		

Week	Statements	Teaching activities	Notes
25	<p>1.09e Understand and be able to show how such methods can fail.</p> <p><i>In particular, learners should know that:</i></p> <ol style="list-style-type: none"> <li>1. the iteration <math>x_{n+1} = g(x_n)</math> converges to a root at <math>x = a</math> if <math> g'(a)  &lt; 1</math>, and if <math>x_1</math> is sufficiently close to <math>a</math>;</li> <li>2. the Newton-Raphson method will fail if the initial value coincides with a stationary point.</li> </ol>		<p>SECTION 3 LIMITATIONS OF NR METHOD Page 307</p> <p>EXERCISE 14C Page 310</p>

Week	Statements	Teaching activities	Notes
26	<p>1.09c Be able to solve equations approximately using simple iterative methods, and be able to draw associated cobweb and staircase diagrams.</p>		<p>SECTION 4 FIXED-POINT ITERATION Page 313</p> <p>EXERCISE 14D Page 316</p>
	<p>1.09d Be able to solve equations using the Newton-Raphson method and other recurrence relations of the form <math>x_{n+1} = g(x_n)</math>.</p>		

Week	Statements	Teaching activities	Notes
27	<p>1.09e Understand and be able to show how such methods can fail.</p> <p><i>In particular, learners should know that:</i></p> <ol style="list-style-type: none"> <li>1. the iteration <math>x_{n+1} = g(x_n)</math> converges to a root at <math>x = a</math> if <math> g'(a)  &lt; 1</math>, and if <math>x_1</math> is sufficiently close to <math>a</math>;</li> <li>2. the Newton-Raphson method will fail if the initial value coincides with a stationary point.</li> </ol>		<p>SECTION 5 LIMITATIONS OF FIXED-POINT ITERATION Page 318</p> <p>EXERCISE 14E Page 323</p>

Week	Statements	Teaching activities	Notes
28			MIXED PRACTICE 14 Page 327-329

Week	Statements	Teaching activities	Notes
29	<p>1.09f Understand and be able to use numerical integration of functions, including the use of the trapezium rule, and estimating the approximate area under a curve and the limits that it must lie between.</p> <p><i>Learners will be expected to use the trapezium rule to estimate the area</i></p>		<p>CHAPTER 15 NUMERICAL INTEGRATION Page 330</p> <p>SECTION 1 INTEGRATION AS THE LIMIT OF A SUM Page 331</p> <p>EXERCISE 15A Page 334</p>

	<p><i>under a curve and to determine whether the trapezium rule gives an under- or overestimate of the area under a curve.</i></p> <p><i>Learners will also be expected to use rectangles to estimate the area under a curve and to establish upper and lower bounds for a given integral. See also 1.08g.</i></p> <p><i>[Simpson's rule is excluded]</i></p>		
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30	<p>1.09f Understand and be able to use numerical integration of functions, including the use of the trapezium rule, and estimating the approximate area under a curve and the limits that it must lie between.</p> <p><i>Learners will be expected to use the trapezium rule to estimate the area under a curve and to determine whether the trapezium rule gives an under- or overestimate of the area under a curve.</i></p> <p><i>Learners will also be expected to use rectangles to estimate the area under a</i></p>		<p>SECTION 2 THE TRAPEZIUM RULE Page 336</p> <p>EXERCISE 15B Page 339</p>



	<p><i>curve and to establish upper and lower bounds for a given integral. See also 1.08g.</i></p> <p><i>[Simpson's rule is excluded]</i></p>		
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<b>Week</b>	<b>Statements</b>	<b>Teaching activities</b>	<b>Notes</b>
31	<p>1.09f Understand and be able to use numerical integration of functions, including the use of the trapezium rule, and estimating the approximate area under a curve and the limits that it must lie between.</p> <p><i>Learners will be expected to use the trapezium rule to estimate the area under a curve and to determine whether the trapezium rule gives an under- or overestimate of the area under a curve.</i></p> <p><i>Learners will also be expected to use rectangles to estimate the area under a curve and to establish upper and lower bounds for a given integral. See also 1.08g.</i></p> <p><i>[Simpson's rule is excluded]</i></p>		MIXED PRACTICE 15 Page 342-344

